HOME WORK

**#1.6,1.7,1.8**

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**#1.6**

6. Use rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.”

**Answer**: Let r be the proposition “It rains,” let f be the proposition “It is foggy,” let s be the proposition “The sailing race will be held,” let l be the proposition “The lifesaving demonstration will go on,” and let t be the proposition “The trophy will be awarded.” We are given premises (¬r ∨ ¬f) → (s ∧ l), s → t, and ¬t. We want to conclude r. We set up the proof in two columns, with reasons, as in Example 6. Note that it is valid to replace subexpressions by other expressions logically equivalent to them.

**Step Reason**

1. ¬t Hypothesis

2. s → t Hypothesis

3. ¬s Modus tollens using (1) and (2)

4. (¬r ∨ ¬f) → (s ∧ l) Hypothesis

5. (¬ (s ∧ l)) → ¬ (¬r ∨ ¬f) Contrapositive of (4)

6. (¬s ∨ ¬l) → (r ∧ f) De Morgan’s law and double negative

7. ¬s ∨ ¬l Addition, using (3)

8. r ∧ f Modus ponens using (6) and (7)

9. r Simplification using (8)

12. Show that the argument form with premises (p ∧ t) → (r ∨ s), q → (u ∧ t), u → p, and ¬s and conclusion q → r is valid by first using Exercise 11 and then using rules of inference from Table 1.

**Answer**: Applying Exercise 11, we want to show that the conclusion r follows from the five premises (p ∧ t) → (r ∨ s), q → (u ∧ t), u → p, ¬s, and q. From q and q → (u ∧ t) we get u ∧ t by modus ponens. From there we get both u and t by simplification (and the commutative law). From u and u → p we get p by modus ponens. From p and t, we get p ∧ t by conjunction. From that and (p ∧ t) → (r ∨ s) we get r ∨ s by modus ponens. From that and ¬s we finally get r by disjunctive syllogism.

14. For each of these arguments, explain which rules of inference are used for each step.

a) “Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”

**Answer**: In each case we set up the proof in two columns, with reasons, as in Example 6. a) Let c(x) be “x is in this class,” let r(x) be “x owns a red convertible,” and let t(x) be “x has gotten a speeding ticket.” We are given premises c(Linda), r(Linda), ∀x(r(x) → t(x)), and we want to conclude ∃x(c(x) ∧ t(x)).

**Step**  **Reason**

1. ∀x(r(x) → d(x)) Hypothesis

2. r(y) → d(y) Universal instantiation using (1)

3. ∀x(d(x) → a(x)) Hypothesis

4. d(y) → a(y) Universal instantiation using (3)

5. r(y) → a(y) Hypothetical syllogism using (2) and (4)

6. ∀x(r(x) → a(x)) Universal generalization using (5)

b) “Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.”

**Answer**: Let r(x) be “r is one of the five roommates listed,” let d(x) be “x has taken a course in discrete mathematics,” and let a(x) be “x can take a course in algorithms.” We are given premises ∀x(r(x) → d(x)) and ∀x(d(x) → a(x)), and we want to conclude ∀x(r(x) → a(x)). In what follows y represents an arbitrary person.

**Step Reason**

1. ∀x(r(x) → d(x)) Hypothesis

2. r(y) → d(y) Universal instantiation using (1)

3. ∀x(d(x) → a(x)) Hypothesis

4. d(y) → a(y) Universal instantiation using (3)

5. r(y) → a(y) Hypothetical syllogism using (2) and (4)

6. ∀x(r(x) → a(x)) Universal generalization using (5)

c) “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.”

**Answer**: Let s(x) be “x is a movie produced by Sayles,” let c(x) be “x is a movie about coal miners,” and let w(x) be “movie x is wonderful.” We are given premises ∀x(s(x) → w(x)) and ∃x(s(x) ∧ c(x)), and we want to conclude ∃x(c(x) ∧ w(x)). In our proof, y represents an unspecified particular movie.

**Step Reason**

1. ∃x(s(x) ∧ c(x)) Hypothesis

2. s(y) ∧ c(y) Existential instantiation using (1)

3. s(y) Simplification using (2)

4. ∀x(s(x) → w(x)) Hypothesis

5. s(y) → w(y) Universal instantiation using (4)

6. w(y) Modus ponens using (3) and (5)

7. c(y) Simplification using (2)

8. w(y) ∧ c(y) Conjunction using (6) and (7)

9. ∃x(c(x) ∧ w(x)) Existential generalization using (8)

d) “There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.”

**Answer**: Let c(x) be “x is in this class,” let f(x) be “x has been to France,” and let l(x) be “x has visited the Louvre.” We are given premises ∃x(c(x) ∧ f(x)), ∀x(f(x) → l(x)), and we want to conclude ∃x(c(x) ∧ l(x)). In our proof, y represents an unspecified particular person.

**Step Reason**

1. ∃x(c(x) ∧ f(x)) Hypothesis

2. c(y) ∧ f(y) Existential instantiation using (1)

3. f(y) Simplification using (2)

4. c(y) Simplification using (2)

5. ∀x(f(x) → l(x)) Hypothesis

6. f(y) → l(y) Universal instantiation using (5)

7. l(y) Modus ponens using (3) and (6)

8. c(y) ∧ l(y) Conjunction using (4) and (7)

9. ∃x(c(x) ∧ l(x)) Existential generalization using (8)

18. What is wrong with this argument? Let S(x, y) be “x is shorter than y.” Given the premise ∃sS(s, Max), it follows that S(Max, Max). Then by existential generalization it follows that ∃xS(x, x), so that someone is shorter than himself.

**Answer**: We know that some s exists that makes S(s, Max) true, but we cannot conclude that Max is one such s. Therefore, this first step is invalid.

**#1.7**

8. Prove that if n is a perfect square, then n + 2 is not a perfect square.

**Answer**: Let n = m2. If m = 0, then n + 2 = 2, which is not a perfect square, so we can assume that m ≥ 1. The smallest perfect square greater than n is (m + 1)2, and we have (m + 1)2 = m2 + 2m +1= n + 2m + 1 > n + 2 · 1+1 > n + 2. Therefore n + 2 cannot be a perfect square

18. Prove that if n is an integer and 3n + 2 is even, then n is even using

a) a proof by contraposition.

b) a proof by contradiction.

**Answer**:

a) We must prove the contrapositive: If n is odd, then 3n + 2 is odd. Assume that n is odd. Then we can write n = 2k + 1 for some integer k. Then 3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1. Thus 3n + 2 is two times some integer plus 1, so it is odd.

b) Suppose that 3n + 2 is even and that n is odd. Since 3n + 2 is even, so is 3n. If we add subtract an odd number from an even number, we get an odd number, so 3n − n = 2n is odd. But this is obviously not true. Therefore, our supposition was wrong, and the proof by contradiction is complete

28. Prove that m2 = n2 if and only if m = n or m = −n.

**Answer**: There are two things to prove. For the “if” part, there are two cases. If m = n, then of course m2 = n2; if m = −n, then m2 = (−n)2 = (−1)2n2 = n2. For the “only if” part, we suppose that m2 = n2. Putting everything on the left and factoring, we have (m + n) (m − n) = 0. Now the only way that a product of two numbers can be zero is if one of them is zero. Therefore, we conclude that either m + n = 0 (in which case m = −n), or else m − n = 0 (in which case m = n), and our proof is complete.

**#1.8**

12. Show that the product of two of the numbers 651000 − 82001 + 3177, 791212 − 92399 + 22001, and 244493 − 58192 + 71777 is nonnegative. Is your proof constructive or nonconstructive? [Hint: Do not try to evaluate these numbers!]

**Answer**: Of these three numbers, at least two must have the same sign (both positive or both negative), since there are only two signs. (It is conceivable that some of them are zero, but we view zero as positive for the purposes of this problem.) The product of two with the same sign is nonnegative. This was a nonconstructive proof, since we have not identified which product is nonnegative. (In fact, a computer algebra system will tell us that all three are positive, so all three products are positive.)

14. Prove or disprove that if a and b are rational numbers, then ab is also rational.

**Answer**: An assertion like this one is implicitly universally quantified—it means that for all rational numbers a and b, ab is rational. To disprove such a statement, it suffices to provide one counterexample. Take a = 2 and b = 1/2. Then ab = 21/2 = √2, and we know from Example 10 in Section 1.6 that √2 is not rational.

30. Prove that there are no solutions in integers x and y to the equation 2x2 + 5y2 = 14.

**Answer** If |y| ≥ 2, then 2x2 + 5y2 ≥ 2x2 + 20 ≥ 20, so the only possible values of y to try are 0 and ±1. In the former case we would be looking for solutions to 2x2 = 14 and in the latter case to 2x2 = 9. Clearly there are no integer solutions to these equations, so there are no solutions to the original equation.

38. Prove or disprove that if you have an 8-gallon jug of water and two empty jugs with capacities of 5 gallons and 3 gallons, respectively, then you can measure 4 gallons by successively pouring some of or all of the water in a jug into another jug.

**Answer**: The solution is not unique, but here is one way to measure out four gallons. Fill the 5-gallon jug from the 8-gallon jug, leaving the contents (3, 5, 0), where we are using the ordered triple to record the amount of water in the 8-gallon jug, the 5-gallon jug, and the 3-gallon jug, respectively. Next fill the 3-gallon jug from the 5-gallon jug, leaving (3, 2, 3). Pour the contents of the 3-gallon jug back into the 8-gallon jug, leaving (6, 2, 0). Empty the 5-gallon jug’s contents into the 3-gallon jug, leaving (6, 0, 2), and then fill the 5-gallon jug from the 8-gallon jug, producing (1, 5, 2). Finally, top off the 3-gallon jug from the 5-gallon jug, and we’ll have (1, 4, 3), with four gallons in the 5-gallon jug.